1

A GENERAL DESCRIPTION OF DESERT 1.0

1.1THE RIVERINE SYSTEM AND ITS CHARACTERISTICS AS TREATED BY DESERT 1.0

In DESERT, a riverine system is organized as a binary tree made of "reaches". Reaches are segments of the rivers forming the system, where hydraulic and water quality properties are in some sense uniform. Points of separation divide reaches from each other. The fact that the system is organized as a binary tree implicates that, although branching is allowed, no meandering or circling is possible. Also, by convention, the root of the binary tree must be the downstream end of the river system. As a consequence, no bifurcation of rivers is allowed. Another restriction on the topology of the system is that, at each confluence point, only two reaches can merge to form a new one.

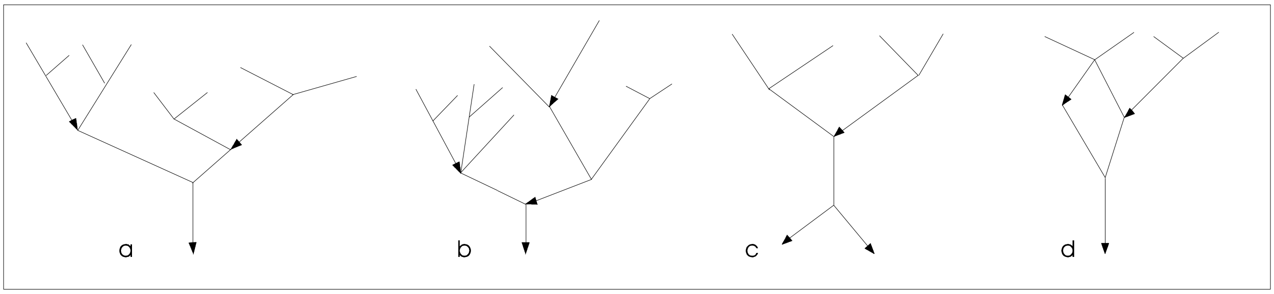


Figure 1.1.1 Admissibility of riverine system schemes.
a) admissible, since it is a binary tree; b) not admissible because showing a confluence with three merging branches; c) not admissible because having two roots and a bifurcation; d) not admissible, because the presence of a cycle within the tree. The arrows indicate the main direction of flow.

The fact that the riverine system is treated as a binary tree implicates that an order is associated to each reach of the system. Adjacent reaches of the same order form a branch of the river system. When two reaches merge at a confluence, one keeps the same order of the downstream reach (therefore it belongs to the same branch of the downstream reach). The other reach increases its order of 1, thus beginning another branch (**figure 1.1.2**).

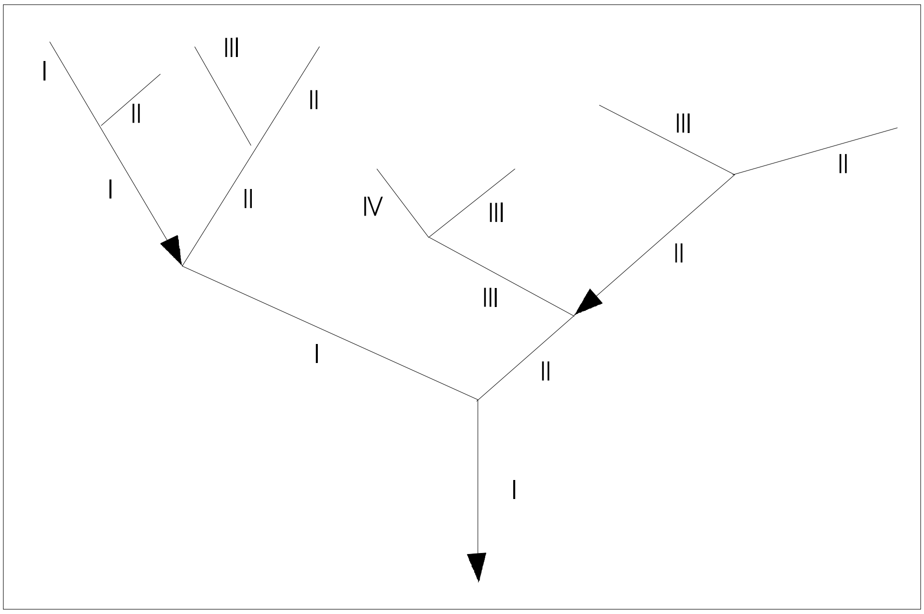
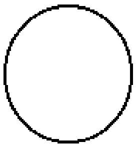


Figure 1.1.2 Representation of a riverine system as a binary tree.
Arrows indicate the main direction of the flow. Numbers indicate the order of reaches.

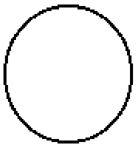
Characteristics of riverine systems (hydraulic sections, confluence of rivers, effluent point sources, lateral inflows, measurement points, etc.) are expressed as a collection of objects belonging to certain classes and situated along the reaches. There are two kinds of objects: **structural** objects and **river** objects.

Structural objects are objects that modify the topology of the hydraulic system. They define the beginning and the end of reaches. They do not have any information record associated to them.
River objects are objects that do not modify the topology of the system, but have one or more information record associated to them. The classes implemented in the version 1.0 of DESERT are described below, together with their symbolic representation used by the display unit (**see Section 1.2.2**):

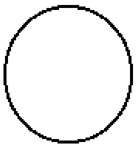
Structural objects



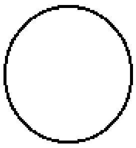
Point of separation of reaches. It indicates the end of a reach and the beginning of a new reach. Reaches are segments of the river "uniform" from the hydraulic and water quality viewpoint. Typically, a reach can be a segment of a river where water discharge is longitudinally uniform, a segment of a river upstream or downstream a load emission, and so on. DESERT automatically generates a Reach cross section object in correspondence of this object. Its shape is obtained by linear interpolation of the two nearest Reach cross section objects present on the same branch. This is done to supply boundary conditions for hydraulic simulation.



Confluence of two reaches. This object indicates that two reaches merge to form one reach only. Water discharges of the merging reaches are summed to produce the water discharge of the reach downstream of the confluence. A Confluence object automatically acts also as a Point of separation object. DESERT automatically generates a Reach cross section object in correspondence of this object. Its shape is obtained by linear interpolation of the two nearest Reach cross section objects present on the same branch. This is done to supply boundary conditions for hydraulic simulation.

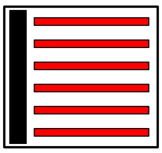


Headwater point. A Headwater object must mark the upstream end of each terminal reach. DESERT automatically generates a Reach cross section object in correspondence of this object. Its shape is obtained by linear interpolation of the two nearest Reach cross section objects present on the same branch. This is done to supply boundary conditions for hydraulic simulation.

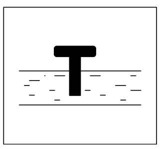


End point. An End object must mark the root of the binary tree representing the system. DESERT automatically generates a Reach cross section object in correspondence of this object. Its shape is obtained by linear interpolation of the two nearest Reach cross section objects present on the same branch. This is done to supply boundary conditions for hydraulic simulation.

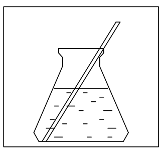
River objects



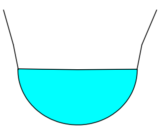
Lateral inflow source. The presence of this object means that there is lateral inflow along the reach where the object is located. The inflow rate is specified in $m^3 s^{-1} Km^{-1}$. One or more water quality records are associated to this object (**see Chapter 3.3**).



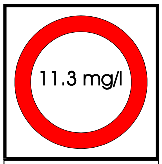
Water intake. This object indicates a water intake from the reach. It can be associated to one or more discharge and water quality records (**see Chapter 3.3**). Discharges are expressed in $m^3\ s^{-1}$ and water quality data in mg l^{-1}.



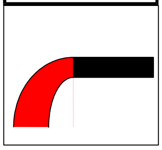
Measurement point. It represents a point where water quality has been measured. It can be used both for representing available data and for calibration purposes. One or more water quality records are associated to this object(**see Chapter 3.3**). Water quality data in mg l^{-1}.



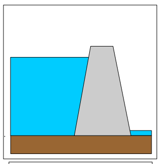
Reach cross section. This object indicates the shape of the reach cross section in a point. A succession of pairs "distance from the longitudinal axis of the reach"-"elevation of the riverbed above mean sea level" describe the cross sectional shape. Since the hydraulic and water quality models are one-dimensional, the shape of the cross-sections is supposed symmetric with respect to the longitudinal axis. For this reason, it is necessary to specify the pairs width-bottom elevation of half the section only. DESERT automatically associates a Reach cross section object to every structural object. Its shape is determined by linearly interpolating the two nearest Reach cross section object of the same branch. Both width and bottom elevation are expressed in meters.



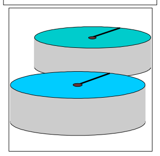
Water quality constraint. This object indicates that water quality should satisfy some constraints at the location of the object. It has associated one or more water quality records expressed in mg l^{-1} (**see Chapter 3.3**). It is used mainly for optimization purposes.



Point Source. This object indicates that some effluent is discharged where the object is located. One or more water quality records describe the quality and quantity of the effluent (**see Chapter 3.3**). Discharges are expressed in m^3 s^{-1} and water quality data in mg l^{-1}.



Weir. This object indicates the presence of a hydraulic constraint to the flow. The constraint can be either a “discharge”-“elevation” relationship or a “water surface elevation”-“time” relationship. (**see Chapter 3.3**).



Treatment plant. This object identifies a treatment plant discharge. Water quality resulting from alternative policies and the relative costs can be associated to this object (**see Chapters 3.2 and 3.3**).

The objects present in the river basin need to be labeled by an identification code. Their logical position must be given in the River Index Code, a system based on the concept of binary tree and of order of the branches (**see Chapter 3.3**).

1.2A GENERAL OVERVIEW OF THE ORGANIZATION OF DESERT 1.0

DESERT 1.0 has been written completely in C++ language to permit the use of the object oriented programming and the possibility of management and upgrade of the code that this language allows.
Figure 1.2.1 depicts the overall scheme of the software. A deeper description of the single sections is given later in the chapter.

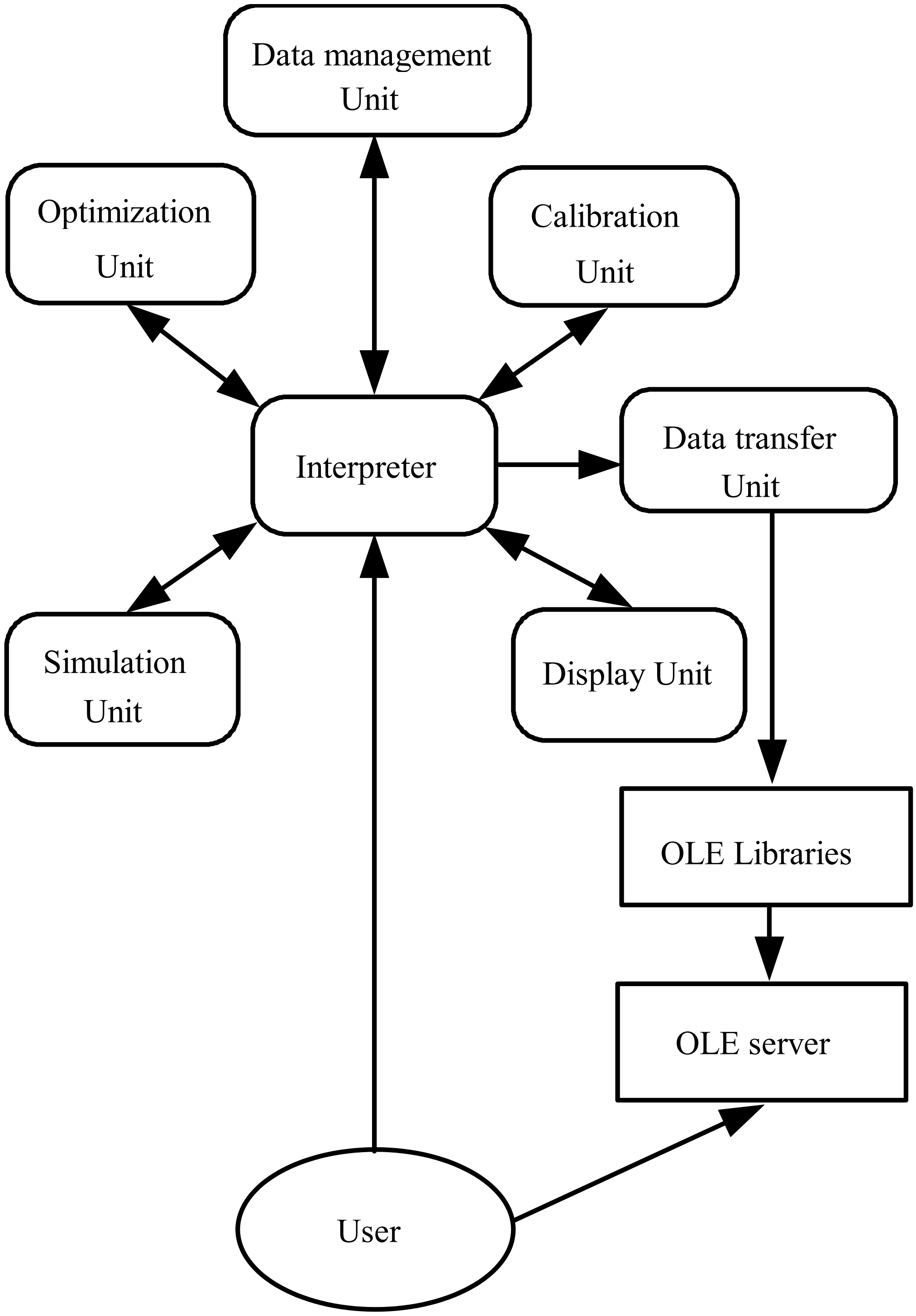


Figure 1.2.3 Outline of the DESERT 1.0 software.

1.2.1Data management unit

One of the main features of DESERT is that it incorporates a powerful dBase style relational database engine to handle input data. This feature allows the user to treat the large amount of heterogeneous data (data on the river network, effluents, treatment plants, monitoring network, etc.) necessary for a river basin scale assessment using any dBase compatible database management software. In this way the user can organize input data in a really effective, flexible, and easy way (**See Chapter 3.3**). The fact that monitoring networks often organize their results using databases is a further element in favor of this choice. Also it is very easy the convention from the output format of traditional programming language (normally text separated by commas or tabs) into dBase. The only limitation of the dBase engine is that it is based on the dBase III language. The use of later versions of this language (as the more common dBase III+ or dBase IV) is possible most of the times. However, since some problems could occur, it should be better to use the version III of dBase.

1.2.2Display unit

As already introduced, river basins consist of spatially distributed objects, such as river stretches, gauging and effluent source points, sampling locations, weirs, etc. Therefore, the best way to represent such a collection of objects is to display a scheme of its spatial distribution. The display unit guarantees this possibility. The display unit draws river basins in a symbolic way and is capable of scaling, scrolling and selecting particular river objects. DESERT can automatically generate the representation of the river basin as a binary tree in a very schematic way (**Figure 1.2.2**). Reaches belonging to the same branch are represented along horizontal or vertical lines. At the confluences, the lower order branches depart from the main order branch perpendicularly. The user can only specify if the lower order branch intersects the right or the left side of the main branch. Vertical representations of vertical branches are proportional to each other. On the contrary, drawing requirements determine the length of every single horizontal branch.

Figure 1.2.4 The automatic representation of a riverine system.

It is also possible to have a representation of the system closer to realty. This can be done by supplying the geographical coordinates of the structural objects as vector files in the format of commercial MapViewer (TM) mapping software from Golden Software, Inc. **(Figure 1.2.3**). This format can be easily produced manually or adapted from other formats if needed (**See Chapter 3.3**). In this case, the length of each reach is proportional to the difference between the geographical coordinates of its boundary points and not to the actual distance.

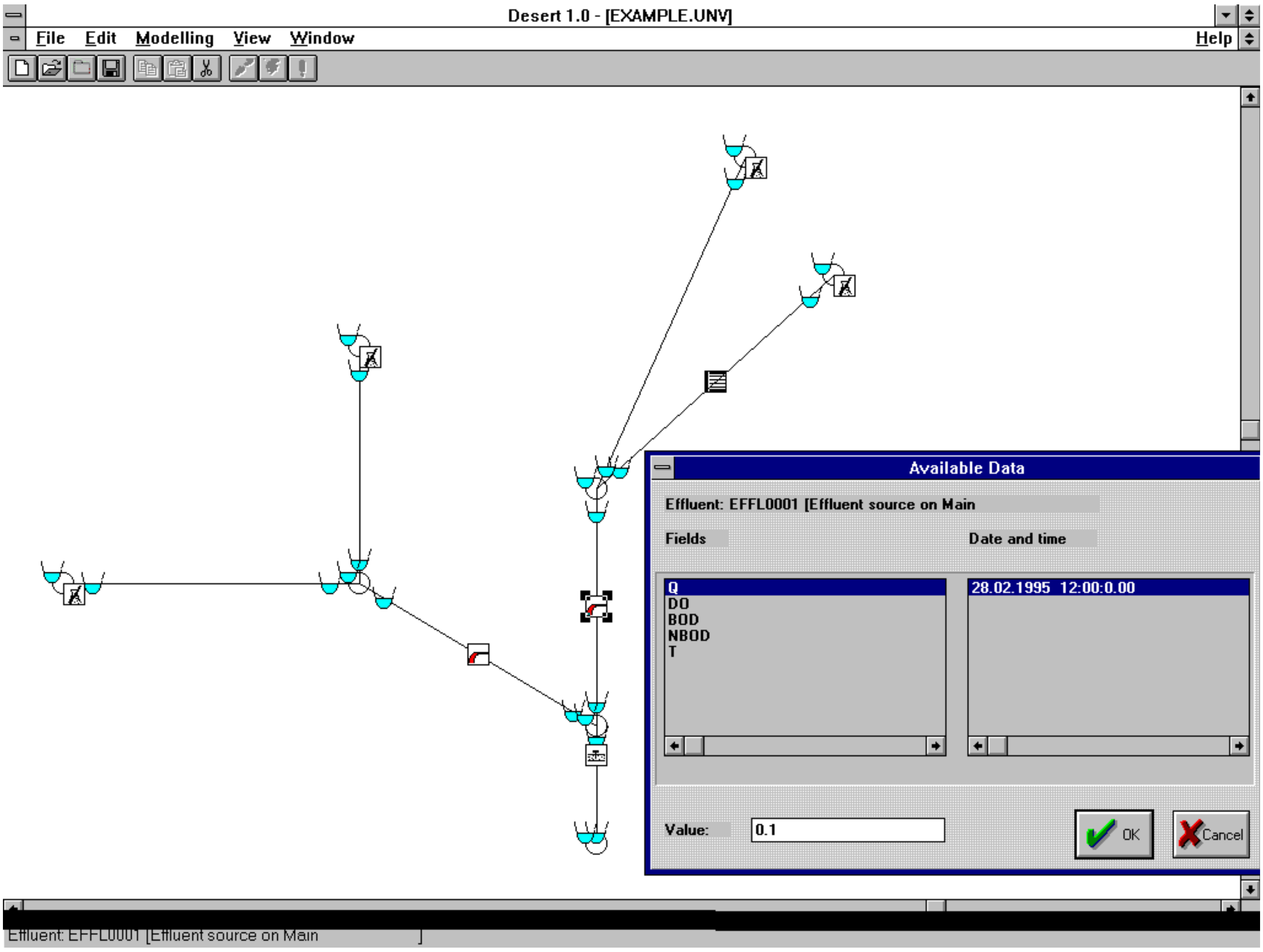


Figure 1.2.5 The representation of a riverine system based on a MapViewer (TM) file.
The box on the lower right corner is displaying data on a source object selected on the window. The properties of any of the objects belonging to the system can be displayed in the same way.

1.2.3Hydraulic unit

Computing river hydraulic parameters, such as depth, cross-section area, travel time, etc., is a necessary precondition for the simulation of water quality. The hydraulic models for rivers and open channels are based on mass continuity and moment equations from fluid mechanics (Antontsev et al., 1986). Most of the hydraulic and water quality simulation packages oblige the user to adopt the same method of solution for all the elements of the network. This restriction makes the software simpler, but does not take into consideration that available data and user's interest vary from reach to reach. On the contrary, DESERT allows the use of a specific hydraulic model for every reach of the river basin. The dialogue box supplied by the display unit (**see Chapter 3.5**) allows the user to choose the hydraulic model out of the five possible alternatives listed below. However, all the five alternatives are based on a one dimensional spatial approach. That is, water velocity is averaged over the cross-section area of the reaches.

None

No hydraulic simulation is performed. Therefore, the reach is considered as not existing.

Water balance (No mesh)

The objects present on the reach subdivide the reach itself into segments. Simple steady state water balance is performed between consecutive segments. Consequently, the hydraulics is described by the continuity equation alone in the following form:

$$Q_d = Q_u + q^+$$

where Q_d is the discharge downstream the object
Q_u is the discharge upstream the object
q^+ is positive if the object discharges water in the reach or negative if it withdraws water from the reach

This method, and the following one, are typically used for reaches where unimportant processes take place, for reaches where hydraulic data are not sufficient, or during early stages in the modeling process.

Water balance (Mesh)

This alternative uses the same kind of water balance performed in the previous case. The difference is that the reach is subdivided into segments of length specified by the user and water balance is not applied from object to object, but from mesh point to mesh point.

$$Q_{n+1} = Q_n + q^+$$

where Q_{n+1} is the discharge at the mesh point n+1
Q_n is the discharge at the mesh point n
q^+ is positive if water is discharged between points n and n+1 or negative if it is withdrawn

Steady state approximation

For a steady-state situation, which is typical of low-flow periods, the momentum and continuity equations can be simplified by omitting terms that are responsible for the dynamic behavior of the flow. A quadratic law of resistance usually represents the friction term in the momentum equation:

$$\frac{\partial h}{\partial x} = - \frac{Q|Q|}{K^2}$$

while continuity equation is given by:

$$\frac{\partial Q}{\partial x} = q^+(x)$$

where x is the co-ordinate along the river or channel
Q is the stream flow rate
h is the local depth of the river or channel
K is the resistance parameter (that in DESERT has a Manning's form)
q^+ is the lateral inflow

Of course, also in this case the reach is subdivided in a series of segments, the length of which is specified by the user through a dialogue box (**see Chapter 3.5**). The solution of this system of ordinary differential equations is carried out with a Modified Euler Method.

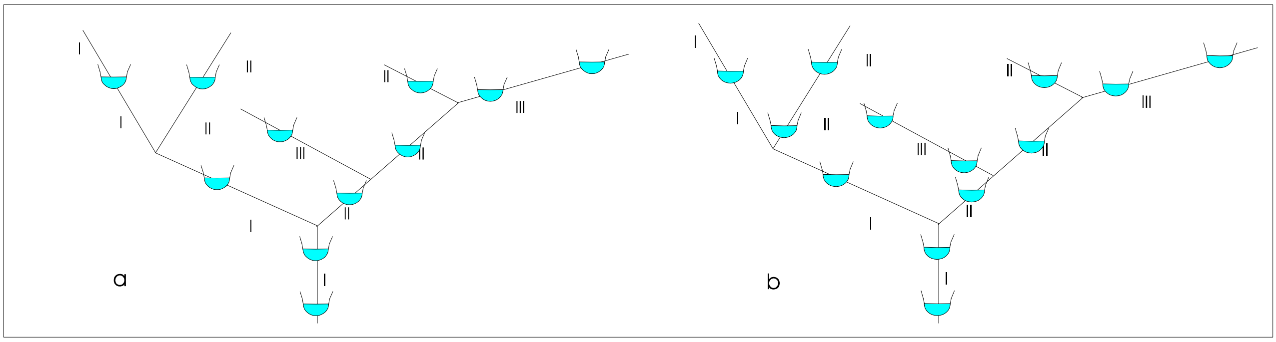


Figure 1.2.6 Requirements for the application of Steady state and Diffusion wave approximations.
a) Steady state or diffusion wave approximation can not be applied to the whole system (the branch of order II on the left and one of the branches of order III on the right have only one section); b) Steady state and diffusion wave approximation can be applied to the whole system (all the branches have at least two cross-section)

A minimum number of two Reach cross section objects is necessary for any branch where steady state or diffusion wave approximations are used. If this requirement is not satisfied, DESERT will not have sufficient information to carry out the hydraulic simulation (**see Figure 1.2.4**). On the branches where this minimum requirement is not satisfied Water balance methods can be applied. The above mentioned is a minimum requirement for using the momentum equation. However, to have a reliable hydraulic simulation, experience points out that a cross section should not be distant from the next cross section more than five times the space equivalent to its width.

Diffusion wave approximation

Since the software is oriented primarily towards water quality management, the level of complexity of hydrological models can be limited. For dynamic situations (non-stationary in time), the diffusion wave approximation (Antontsev et al., 1986) strikes a good balance between complexity, accuracy, and computation speed:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q^+(x, t)$$

$$\frac{\partial z}{\partial x} = - \frac{Q|Q|}{K^2}$$

where z is the water surface elevation

q^+ -is the water discharge source along the river stretch
A is the cross-sectional area along the reach
t is the time variable

The method of solution of this couple of partial differential equation is a fully implicit scheme with Newtonian iterations. Steady state simulation with the initial boundary conditions determines the initial condition for the dynamic method. The boundary condition normally adopted for the downstream nodes is

$$\frac{\partial z}{\partial x} = 0$$

However, if the downstream node is a Weir, then a Q(z) relationship or a z(t) relationship can also be imposed.

1.2.4Water quality simulation unit

Water quality assessment in riverine systems is generally carried out solving a set of ordinary or partial differential equations, which describe relevant physical and chemical processes. Most of the water quality software packages implement only one method of solution of the transport equation for all the network. Often, they are also problem specific (i.e., they deal with eutrophycation or with heavy metals, or with pesticide, etc.). Also they often oblige the user to adopt one formulation of specific processes (light extinction, temperature dependence, photosynthetic rates, and so on) among the ones supported by the software library. Alternatively, the modeler must code the model in some computer language, as FORTRAN, C, or PASCAL, build the executable module with the help of tools like compilers, linkers, librarians, etc., and run it. The process is iterative, since, after evaluation of results, corrections may be needed to programming script or model formulation. The overall process is very slow and prone to errors (**see Figure 1.2.5**).

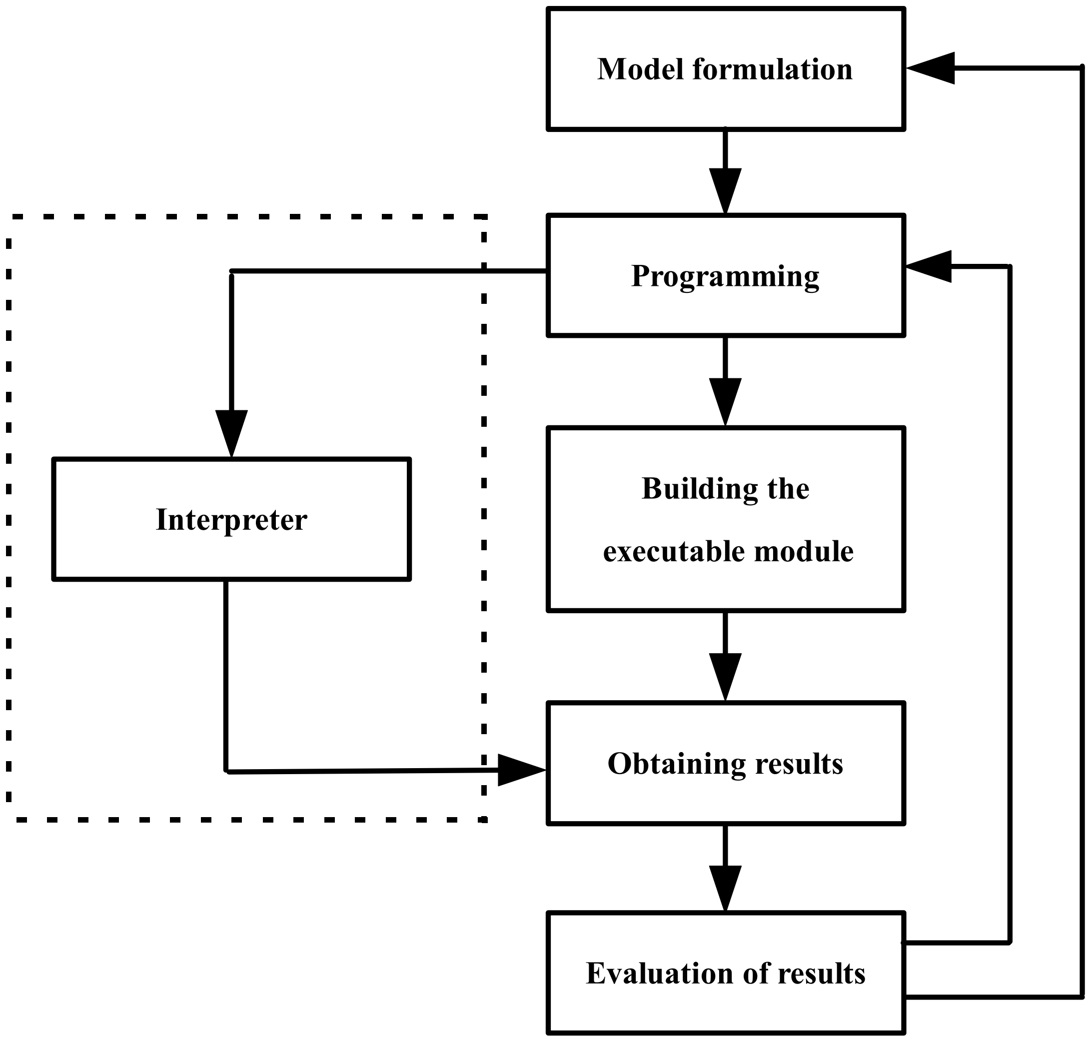


Figure 1.2.7 Conventional modeling approach (right) versus DESERT approach (left, in the dashed box).

DESERT represents an alternative to both the previously illustrated approaches. First, it supplies five different ways to simulate mass balance equations that may coexist in the same river basin schematization. Second, and more important, for three of these approaches (Matter kinetics (No mesh), Matter transport (Steady state solution), and Matter transport (Dynamic solution)) description of reaction processes can be given through an interpreting language similar to BASIC. However, to speed up the simulation process, the transport part of the equations is precompiled. In fact, a mass conservation equation for riverine systems can be expressed as follows:

$$\frac{\partial(A \cdot C_i)}{\partial t} + \frac{\partial(Q \cdot C_i)}{\partial x} = q^+(x,t) \cdot C_i^+(x,t) + F_i(x,t,C_1,C_2,..,C_i,..,C_N)$$

i=1,...,N

where: C_i are the water quality constituents (state variables)

N is the number of state variables

C_i^+ is the concentration of i[th] water quality constituent in source water discharges

F_i is the function that expresses the reactions scheme of the i[th] state variable

Water quality equations can be decomposed into two sections:

1. the left hand side, which is the generic part of the equation (transport equation) and that is computed using precompiled instructions;
2. the right hand side, which is the specific part of the equation (reaction scheme) that is computed using interpreted commands.

This approach, together with the way input data are treated, allows the user to be free in describing whatever dynamics he (or she) is interested on. That is, the user can specify as many variables as desired and define whatever reaction may be needed. However, the user should note that among the arguments of the function $F_i(x, t, C_1, C_2, ., C_i, .., C_N)$ derivatives do not appear. This is the only limitation to the formulation of the right hand side of the transport equations. The most important consequence of this limitation is that longitudinal dispersion is not considered. On the other hand, in most of the riverine systems, advection is the dominating factor of transport phenomena (Sómlyódy and Varris, 1992).

In specifying the reaction schemes of the model and in determining the state variables, the user must consider that the only one water quality model is applied to all the reaches composing the system. Only the method for solving the mass balance equation can vary from reach to reach.

Another aspect to be considered is that all the six alternative water quality formulations are based on a one dimensional spatial approach. That is, water quality variables are averaged over the cross-sectional area of the reaches. The method for solving the water quality equations implemented in DESERT are the following:

None

No water quality simulation is performed and the reach is considered as not existing from the water quality view point.

Matter balance (No mesh)

Simple steady state mass balance is performed on the segments of the reach delimited by the objects present on the reach itself. The element concentrations are described by continuity equation alone in the following form:

$Q_d C_d = Q_u C_u + q^+ C^+$

where Q_d is the discharge downstream the object

Q_u is the discharge upstream the object

q^+ is positive if the object discharges water in the reach or negative if it withdraws water from the reach.

C_d is the concentration downstream

C_u is the concentration upstream

C^+ is the inflow concentration.

This method, and the following one, are typically used for the reaches where unimportant processes take place, for reaches where water quality data are not satisfying, or during early stages in the modeling process.

Matter balance (Mesh)

This alternative uses the same kind of matter balance performed in the previous case. The difference with the previous case is that the reach is subdivided into segments of length specified by the user (the same used for the solving hydraulics, if a meshed method was chosen). Then, water balance is not applied from object to object, but from mesh point to mesh point.

$$Q_{n+1}C_{n+1} = Q_nC_n + q^+C^+$$

where Q_{n+1} is the discharge at the mesh point n+1
Q_n is the discharge of the mesh point n
q^+ is positive if water is discharged between points n and n+1 or negative if it is withdrawn
C_{n+1} is the concentration at the point n+1
C_n is the concentration at the point n
C^+ is the inflow concentration

Matter kinetics (No mesh)

With this option, no differential or transport equation is solved. The user directly specifies the desired kinetics of variables. Exponential, Bessel type, ERF type, and other kinds of behavior can be used for describing element kinetics. Functions are applied in the segments between objects. The contribution of inflow or withdrawn to the mass balance is calculated at the object positions. The selected behavior is then applied to the new concentration, i.e.:

Along the segments

$$C_i(x) = F_i(x, C_1, C_2, .., C_i, .., C_N)$$

At an object of position x_0:

$$Q(x_0^+)C(x_0^+) = Q(x_0^-)C(x_0^-) + q^+C^+$$

where x_0^+ is a position immediately downstream the object
x_0^- is a position immediately upstream the object.

Matter transport (Steady state approximation)

The steady state approximation of transport equations solves the following equation:

$$\frac{\partial(Q \cdot C^i)}{\partial x} = q^+(x) \cdot C_i^+(x) + F_i(x, C_1, C_2, .., C_i, .., C_N)$$

The reach is subdivided in a mesh of points. The mesh size is introduced by the user (**see Chapter 3.5**). The numerical method used to solve the equation is a Runge-Kutta order IV.

Matter transport (Dynamic approximation)

A set of full transport equations is solved in this method for the points of a mesh of user defined dimension (**see Chapter 3.5**). The initial values are determined either by interpolating input data files' values or by running the Matter transport (Steady state solution) methods with the initial boundary conditions. The solution of the system of equations is obtained by an implicit 1^{st} order running wave method.

1.2.5Data transfer unit

Simulation results have to be presented in some suitable way: as table, chart, or plot. It is not a simple task, since simulation data are usually multidimensional. Moreover, results often must undergo some postcomputational processing, such as statistical analysis or curve fitting. Commercial spreadsheet packages such as Microsoft Excel, Lotus 1-2-3, Corel Chart, etc., can handle this task very easily. The traditional approach of water quality and hydraulic simulation packages has been to produce results in a text file form, with the fields separated from each other by commas, tabs, or spaces. These text files were later processed by some spreadsheet or statistical package, a procedure slow and prone of errors of conversion. Thanks to the Microsoft Windows 3.1 data transfer mechanism based on OLE (Object Linking and Embedding) protocol, today it is possible to establish linkages between applications. Thus, DESERT software makes use of OLE server applications and can save and plot results directly to a spreadsheet. Although theoretically any spreadsheet implementing OLE protocol could be used, in practice only Microsoft Excel is able to satisfactorily handle data produced by DESERT. Data transfer unit facilitates transfer of simulation data through OLE libraries to Microsoft Excel, where data can be independently processed, stored, plotted, and so on.

1.2.6Calibration unit

Calibration procedure is especially important in water quality management, since in the state-of-the-art approach all the uncertainty associated with the modeling process is treated as parameter uncertainty. A number of methods have been developed for parameter estimation and uncertainty analysis in water quality modeling. Most of them use minimization procedures with corresponding loss function such as the least squares methods (Beck, 1979a). Recursive methods, such as Kalman filter, have been also applied (Beck, 1979b; Rinaldi et al., 1979).

All these traditional methods produce a single "best" set of parameter values that is then used for the simulation. Recent developments pointed out that the estimation of a single set of parameters is not able to account for all the uncertainty peculiar to water quality modeling. The stochastic methodology for parameter estimation currently implemented in the software is the Hornberger-Spear-Young behavior definition method (Hornberger et al., 1980). This technique generates a set of parameter vectors on the basis of a pre-specified random distribution for each parameter. A simulation of the model is accomplished for each parameter vector. The model performance is then judged on the basis of knowledge of the system, or "behavior definition" (e.g., lower and upper bounds of state variables, distance of model trajectory from the observed values, etc.). This knowledge can be vague, allowing explicit incorporation of large uncertainties in the calibration procedure. With this technique is possible to account for uncertainties on model parameters and water quality measurements. Other inherent uncertainties (such as uncertainty in temperature, river streamflow etc.) could be taken into account in a sensitivity fashion (**see Chapter 3.8**).

1.2.7Optimization unit

Usually, water quality management problems can be formulated as a search for suitable regional wastewater treatment policies. Many techniques have been applied to identify optimal water quality control strategies. The most commonly used are linear programming, dynamic programming, and simulation models. Non linear programming has been used more rarely due to its complexity and relevant computation requirements. DESERT uses dynamic programming (DP) technique (Bellman, 1957) for accomplishing the optimal allocation of waste water treatment plants. This technique decomposes a problem with a sequence of decisions into a cascade of subproblems, each having one or a reduced number of decisions. These subproblems are resolved recursively, by considering the sub-optimal solution(s) of one subproblem as input(s) to the subsequent subproblem.
The selection of optimal wastewater treatment alternatives in a river basin is a temporally and spatially sequential decision problem. Spatially, decisions are taken for a series of locations in a river basin. Due to the downstream propagation of pollutants of rivers, the water quality at a particular location along a river is entirely determined by the water quality at the immediate upstream discharge/control point (or at several discharge/control points in the case the location is below confluences). Similarly, when investigating investments over a temporal horizon, managers take decisions at points in time. Decisions taken at one time directly affect those to be taken at the next time step.
Clearly, the sequential attributes of DP make this approach very suitable for river water quality management.
Further, DP is generic - the solution algorithm does not depend on complexity of the model, linearity or non-linearity, number of state variables, etc.
One of the most usual formulation for allocating wastewater treatment plants is the following:

$$\min_{x_i \in \Omega_i} \sum_i C_i(X_i)$$

subject to: $$WQ_{j,k}(x_1, \ldots, x_n) \geq WQS_{j,k} \quad \forall j,k$$

where $C_i(x_i)$ is the cost function associated to the alternatives x_i
Ω_i is the set of possible alternatives for the object i
$Wq_{j,k}(x_1,..,x_n)$ is the value of the k^{th} component of the water quality model produced by the set of alternatives $x_1,...,x_n$ at the water quality constraint j
$WQS_{j,k}$ is the constrain on the component k of the water quality model to be satisfied at the water quality constraint j

Since the riverine system is treated as a binary tree, it is always possible to order the reaches as if they were located along a line starting from the most upstream point of the network and proceeding downstream. Therefore, it is possible to design a sequence of discharge/control points suitable for the DP approach. The water quality at the most upstream point is known (because it is a boundary condition), which implies that only one water quality state can be defined. A water quality state at the beginning of a reach is given by the value that components assume at that location. Of course, only the components that must satisfy some water quality constraint characterize states. The water quality resulting at the beginning of the downstream reach is estimated by using model predictions. The same procedure is then applied to the downstream reach, and so on proceeding downstream. When this procedure arrives to a Treatment plant object, the number of possible water quality states increases. The number of water quality states downstream of a treatment plant is given by the product of the water quality states immediately upstream of the treatment plant times the number of alternatives characterizing the treatment plant itself. At a Water quality constraint point, the number of water quality states can be reduced by discarding the ones that do not meet the water quality requirements.
A screening operation at the beginning of each reach limits the growth of possible water quality states. States similar to others, but produced by a set of wastewater treatment alternatives having a higher value of the objective function, are discarded, and only the "cheapest" configurations are preserved for continuing the computation. Two states are considered similar when the distance (i.e., the difference in the value of components) between them is less than a specified quantity.
When the procedure reaches the root of the system (i.e., the End object), the set of feasible solutions (i.e., the solution satisfying the water quality requirements) is completed. Among these solutions, the one produced by the set of wastewater treatment alternatives with the smallest value of the objective function is the optimal solution. It is also possible to print all the feasible solutions in order to check the characteristics of the sub-optimal solutions.

This procedure is mathematically described by the following recursive form:

for the most upstream node

$$f_1(WQ_1) = \min_{\substack{x_1 \in \Omega_1 \\ WQ_1 \geq WQS_1}} C_1(x_1)$$

$$WQ_1 = M_1(x_1, WQ_0)$$

for the n[th] structural object

$$f_n(WQ_n) = \min_{\substack{x_n \in \Omega_n \\ WQ_n \geq WQS_n}} (C_n(x_n) + f_{n-1}(WQ_{n-1}))$$

$$WQ_n = M_n(x_n, WQ_{n-1})$$

where $f_n(\ldots)$ is the objective function at the n^{th} structural object

$f_{n-1}(\ldots)$ is the objective function at the $n-1^{th}$ structural object

$C_n(x_n)$ is the cost function associated to the management alternatives for the n^{th} structural object (0 if no alternative is available)

Ω_n is the set of the possible management alternatives for n^{th} structural object

WQ_n is the water quality at the downstream end of the n^{th} reach

WQ_0 is the water quality at the most upstream node

WQS_n is the minimum water quality to be satisfied at the downstream end of the n^{th} reach (0 if there is no requirement)

$M_n(x_n, WQ_{n-1})$ is the model prediction for the downstream end of the n^{th} reach based on the water quality (WQ_{n-1}) and on the management alternative (x_n)at the upstream end of the reach.

The main drawback of DP technique is the rapid increase in memory requirements as the number of state variables increases (“curse of dimensionality”). Virtual memory in Windows 3.1 is a partial solution to this problem, but at the expense of computational speed.